Arithmetic Sequences

Section 3.5
What’s on the Agenda

- Vocabulary
- Identifying arithmetic sequences
- Finding the next term of an arithmetic sequence
- Finding the $n^{th}$ term of an arithmetic sequence
- Arithmetic sequences and functions
Vocabulary

• A **sequence** is a set of numbers.
• The **terms of a sequence** are the specific order of the numbers in a set.
• An **arithmetic sequence** is a set of numbers where the difference between two successive terms is constant.
• The difference between the two successive terms is known as the **common difference** denoted \(d\).
Identifying an Arithmetic Sequence

- Is the sequence of numbers an arithmetic sequence?
  
  0, 2, 4, 6, 8, 10, …

- The number 0 is the first term, 2 is the second term, 4 is the third term etc.

- The difference between the 1st term and 2nd term, 2 – 0 = 2

- Because the difference between any two successive terms is 2, the sequence has a common difference $d = 2$. So the sequence is an arithmetic sequence!
Identifying an Arithmetic Sequence

• Determine if the sequence is an arithmetic sequence.

\[-\frac{1}{2}, -\frac{3}{8}, -\frac{1}{8}, 0, \frac{1}{4}, \ldots\]

• The difference between the first and second term is $\frac{1}{8}$. The difference between the second and third term is $\frac{2}{8}$ or $\frac{1}{4}$. Since the two differences are not equal this is not an arithmetic sequence.
Identifying an Arithmetic Sequence

• Determine if the sequence is an arithmetic sequence and explain why.
  -20, -16, -12, -8, -4, …
Identifying an Arithmetic Sequence

• Determine if the sequence is an arithmetic sequence and explain why.
  -20, -16, -12, -8, -4,…
• The difference between any two terms is 4.
• So the common difference d=4
• Since the difference between the terms in the sequence is constant this is an arithmetic sequence.
Finding the Next Term

• If a sequence of numbers has a common difference $d$ we can add $d$ to the previous term to get the next term.
• What are the 6th and 7th terms of the arithmetic sequence, 
  \[-13, -8, -3, 2, 7, \ldots\]
• Step 1) Find the common difference $d$
• Step 2) Add $d$ to the 5th term to get the 6th term
• Step 3) Add $d$ to the 6th term to get the 7th term
Finding the Next Term

• What are the 6th and 7th terms of the arithmetic sequence, -13, -8, -3, 2, 7, …

• Step 1) Find the common difference d
  • d=5

• Step 2) Add d to the 5th term to get the 6th term
  • The 5th term is 7 so 7+5=12
  • 6th term is 12

• Step 3) Add d to the 6th term to get the 7th term
  • 12+5=17
  • 7th term is 17
Find the Next Term

- What are the next three terms in the arithmetic sequence?
  -8.5, -9, -9.5, -10,...
Find the Next Term

- What are the next three terms in the arithmetic sequence?
  -8.5, -9, -9.5, -10,…
- d=-0.5
- The 5\textsuperscript{th} term is -10.5
- The 6\textsuperscript{th} term is -11
- The 7\textsuperscript{th} term is -11.5
Finding the $n^{th}$ Term

• So far we have figured out how to find next term from the sum of the previous term and the common difference.
• We can actually find what any term is from the first term and the common difference!
• To find the $n^{th}$ term of any arithmetic sequence with the first term $a_1$ and the common difference $d$ we can use the function

$$a_n = a_1 + (n - 1)d$$

where $n$ is a positive integer
Where does this equation come from!?

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>In Terms of $a_1$ and $d$</th>
<th>In Terms of Successive Terms and $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st term</td>
<td>$a_1$</td>
<td>$a_1$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>2nd term</td>
<td>$a_2$</td>
<td>$a_1 + d$</td>
<td>$a_1 + d$</td>
</tr>
<tr>
<td>3rd term</td>
<td>$a_3$</td>
<td>$a_1 + 2d$</td>
<td>$(a_1+d) + d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$=a_2 + d$</td>
</tr>
<tr>
<td>4th term</td>
<td>$a_4$</td>
<td>$a_1 + 3d$</td>
<td>$(a_2+d) + d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$=a_3 + d$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$n^{th}$ term</td>
<td>$a_n$</td>
<td>$a_1 + (n - 1)d$</td>
<td>$(a_{(n-2)}+d) + d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$=a_{(n-1)} + d$</td>
</tr>
</tbody>
</table>
Finding the $n^{\text{th}}$ Term

- Write an equation for the $n^{\text{th}}$ term of the arithmetic sequence 
  $-12, -8, -4, 0, \ldots$

- Step 1) Find the common difference
  - 4

- Step 2) Write an equation
  - $a_n = a_1 + (n - 1)d$
  - $a_n = -12 + (n - 1)4$
  - $a_n = -12 + 4n - 4$
  - $a_n = 4n - 16$
Finding the $n^{th}$ Term

- Find the 9$^{th}$ term of the sequence.
  -12, -8, -4, 0, …

- Substitute 9 for $n$ in the formula

\[ a_n = 4n - 16 \]

\[ a_9 = 4(9) - 16 \]

\[ a_9 = 20 \]
Finding the $n^{th}$ Term

- Graph the first five terms of the sequence.

-12, -8, -4, 0, ...

<table>
<thead>
<tr>
<th>$n$</th>
<th>$4n-16$</th>
<th>$a_n$</th>
<th>$(n, a_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4(1)-16</td>
<td>-12</td>
<td>(1, -12)</td>
</tr>
<tr>
<td>2</td>
<td>4(2)-16</td>
<td>-8</td>
<td>(2, -8)</td>
</tr>
<tr>
<td>3</td>
<td>4(3)-16</td>
<td>-4</td>
<td>(3, -4)</td>
</tr>
<tr>
<td>4</td>
<td>4(4)-16</td>
<td>0</td>
<td>(4, 0)</td>
</tr>
<tr>
<td>5</td>
<td>4(5)-16</td>
<td>4</td>
<td>(5, 4)</td>
</tr>
</tbody>
</table>
How to Graph on Desmos

(n, a_n)  
(1, -12)  
(2, -8)  
(3, -4)  
(4, 0)  
(5, 4)
Finding the $n^{th}$ Term

• What term of the sequence is 32?

\[ a_n = 4n - 16 \]

\[ 32 = 4(n) - 16 \]

\[ 32 + 16 = 4(n) - 16 + 16 \]

\[ 48 = 4(n) \]

\[ 12 = n \]

• 32 is the 12$^{th}$ term of the sequence.
Finding the n\textsuperscript{th} Term

• Consider the arithmetic sequence
  \[3, -10, -23, -36, \ldots\]
• 1) Write an equation for the n\textsuperscript{th} term of the sequence
• 2) Find the 15\textsuperscript{th} term in the sequence.
• 3) Graph the first 5 terms of the sequence.
• 4) Which term of the sequence is -114?
Finding the $n^{\text{th}}$ Term

- Consider the arithmetic sequence
  
  \[3, -10, -23, -36, \ldots\]

- 1) Write an equation for the $n^{\text{th}}$ term of the sequence
- $d = -13$

  \[a_n = a_1 + (n - 1)d\]

  \[a_n = 3 + (n - 1)(-13)\]

  \[a_n = -13n + 16\]
Finding the $n^{th}$ Term

- Consider the arithmetic sequence
  
  $3, -10, -23, -36, \ldots$

- 2) Find the $15^{th}$ term in the sequence.
  
  $$a_n = -13n + 16$$
  $$a_{15} = -13(15) + 16$$
  $$a_{15} = -195 + 16$$
  $$a_{15} = -179$$

- The $15^{th}$ term of the sequence is $-179$. 
Finding the $n^{\text{th}}$ Term

- Consider the arithmetic sequence

\[3, -10, -23, -36, \ldots\]

- 3) Graph the first 5 terms of the sequence.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$-13n+16$</th>
<th>$a_n$</th>
<th>$(n, a_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-13(1)+16</td>
<td>3</td>
<td>(1,3)</td>
</tr>
<tr>
<td>2</td>
<td>-13(2)+16</td>
<td>-10</td>
<td>(2,-10)</td>
</tr>
<tr>
<td>3</td>
<td>-13(3)+16</td>
<td>-23</td>
<td>(3,-23)</td>
</tr>
<tr>
<td>4</td>
<td>-13(4)+16</td>
<td>-36</td>
<td>(4,-36)</td>
</tr>
<tr>
<td>5</td>
<td>-13(5)+16</td>
<td>-49</td>
<td>(5,-49)</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c|c}
 x & y \\
1 & 3 \\
2 & -10 \\
3 & -23 \\
4 & -36 \\
5 & -49 \\
\end{array}
\]

\[y = -13x + 16\]
Finding the $n^{\text{th}}$ Term

- Consider the arithmetic sequence
  $3, -10, -23, -36, \ldots$

- 4) Which term of the sequence is $-114$?

\[
-114 = -13n + 16 \\
-114 - 16 = -13n + 16 - 16 \\
-130 = -13n \\
10 = n
\]

- $-114$ is the $10^{\text{th}}$ term in the sequence.
Arithmetic Sequences and Functions

- From the graph of an arithmetic sequence we see that arithmetic sequences are linear functions.
- $n$ is the x-value or independent variable
- $a_n$ is the y-value or dependent variable
- $d$ the common difference is the slope.
- $a_n = a_1 + (n - 1)d$ as a linear functions is

$$f(n) = (n - 1)d + a_1$$
Arithmetic Sequences as Functions

- Marisol is mailing invitations to her quinceañera. The arithmetic sequence $0.42, 0.84, 1.26, 1.68, \ldots$ represents the cost of postage.
- 1) Write a function to represent this sequence.
- 2) Graph the function
Arithmetic Sequences as Functions

• Marisol is mailing invitations to her quinceañera. The arithmetic sequence $0.42, 0.84, 1.26, 1.68,...$ represents the cost of postage.

• 1) Write a function to represent this sequence.

• $f(n) = (n - 1)d + a_1$
• $f(n) = (n - 1)0.42 + 0.42$
• $f(n) = 0.42n$
Marisol is mailing invitations to her quinceañera. The arithmetic sequence $0.42, 0.84, 1.26, 1.68, \ldots$ represents the cost of postage.

2) Graph the function

<table>
<thead>
<tr>
<th>n</th>
<th>$0.42n$</th>
<th>f(n)</th>
<th>(n, f(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.42(1)</td>
<td>0.42</td>
<td>(1, 0.42)</td>
</tr>
<tr>
<td>2</td>
<td>0.42(2)</td>
<td>0.84</td>
<td>(2, 0.84)</td>
</tr>
<tr>
<td>3</td>
<td>0.42(3)</td>
<td>1.26</td>
<td>(3, 1.26)</td>
</tr>
<tr>
<td>4</td>
<td>0.42(4)</td>
<td>1.68</td>
<td>(4, 1.68)</td>
</tr>
<tr>
<td>5</td>
<td>0.42(5)</td>
<td>2.10</td>
<td>(5, 1.90)</td>
</tr>
</tbody>
</table>
Is this arithmetic sequence a direct variation?
Review

• All arithmetic sequences have a ________ which is denoted ___.
• The common difference can be either a positive or negative number. (true or false)
• Arithmetic sequences are always direct variations. (true or false)
• You can find any term of an arithmetic sequence by ______ the common difference to the previous term.
• The only things we need to determine the n\text{th} term of a sequence are the _____ term denoted $a_1$ and the common difference.
THE BIG PICTURE

LINEAR FUNCTIONS

y = mx + b
ax + by = c

Direct Variation
y = kx and contains (0,0)

Arithmetic Sequences
\[ a_n = (n - 1)d + a_1 \]
where n is positive